LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – NOVEMBER 2011

MT 5508/MT 5502 - LINEAR ALGEBRA

Date : 08-11-2011 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

PART-A

Answer ALL questions: **(10X2 =20)**

1.) If V is a vector space over a field F, Show that $(-a)v = a(-v) = -(av)$ for $a \in F, v \in V$.

2.) Show that the vectors $(1,1)$ and $(-3,2)$ in $R^2$are linearly independent over , the field of real numbers.

3.) If C is the vector space of the field of complex numbers over the field of real numbers, prove that $dim\ C = 2$.

4.) Define the kernel of a vector space homomorphism.

5.) If V is an inner product space, then prove that $< u, \alpha v + \beta w > = \overline{\alpha} < u, v > + \overline{\beta} < u, w >$ $for\ all\ u, v, w \in V\ and\ \alpha, \beta \in F$.

6.) Define the characteristic roots and characteristic vectors of a linear transformation.

7.) Define Skew-symmetric matrix. Give an example.

8.) If A and B are Hermitian, show that $AB - BA$ is Skew- Hermitian.

9.) Find the rank of the matrix $A = \begin{pmatrix} 1 & 5 & -7 \\ 2 & 3 & 1 \end{pmatrix}$ over the field of rational numbers.

10.) If $T \in A(V)$ is Hermitian, then prove that all its eigen values are real.

PART-B

Answer any FIVE questions: **(5X8=40)**

11.) Show that a non empty subset W of a vector space V over a field F is a subspace of V if and only if W is closed under addition and scalar multiplication.

12.) If S and T are subsets of a vector space V over F then prove the following:

i.) $S \subseteq T$ implies that $L(S) \subseteq L(T)$

ii.) $L(L(S)) = L(S)$

iii.) $L(S \cup T) = L(S) + L(T)$.

13.) If V is a vector space of dimension n , then prove that any set of n linearly independent vectors of V is a basis of V.

14.) Let V and W be two n-dimensional vector spaces over F. Then prove that any isomorphism T of V onto W maps a basis of V onto basis of W.

15.) Prove that for any two vectors $u, v \ in \ V$, $\| u + v \| \leq \| u \| + \| v \|$.

16.) If $\lambda \in F$ is an eigen value of $T \in A(V)$, then prove that for any polynomial $f(x) \in F[x]$, $f(\lambda)$ is an eigen value of $f(T)$.

17.) Show that any square matrix A can be expressed uniquely as the sum of a Symmetric matrix and a Skew-symmetric matrix.

18.) Solve the system of linear equations

$$x_1 + 2x_2 + 2x_3 = 5,$$

$$x_1 - 3x_2 + 2x_3 = -5,$$

$$2x_1 - x_2 + x_3 = -3$$

over the rational field by working only with the augmented matrix of the system.

PART- C

Answer any TWO questions: **(2X20=40)**

19.) a.) Prove that the vector space V over F is a direct sum of two of its subspaces W_1 and W_2 if and only if $V = W_1 + W_2$ and $W_1 \cap W_2 = \{0\}$.

b.) If V is a vector space of finite dimension and W is a subspace of V, then prove that $\dim {}^{V}/_{W} = \dim V - \dim W$.

20.) a.) If U and V are vector spaces over F, and if T is a homomorphism of U onto V with kernel W, then prove that ${}^{U}/_{W} \simeq V$.

b.) If V is a finite - dimensional inner product space and if W is a subspace of V, then prove that $V = W \oplus W^{\perp}$.

21.) Prove that every finite - dimensional inner product space has an orthonormal set as a basis.

22.) a.)Show that if $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not zero.

b.) Find the rank of the matrix

$$A = \begin{pmatrix} 0 & -1 & 3 & -1 & 0 & 2 \\ -1 & 1 & -2 & -2 & 1 & -3 \\ 1 & -2 & 5 & 1 & -1 & 5 \end{pmatrix}$$